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More on supersymmetric D-branes in type IIB plane wave background

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Abstract: We extend the study on D-branes in the type IIB plane wave background to less supersymmetric configurations. We show that many new supersymmetric D-branes can be found by turning on electric as well as magnetic background fluxes, or constantly boosting D-branes.

Keywords: D-branes, Penrose limit and pp-wave background.

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1. Introduction

It has been known that the type IIB string theory admits three maximally supersymmetric backgrounds: flat Minkowski space, $AdS_5 \times S^5$ and gravitational plane wave. The last one was recently known with the discovery [1] that the type IIB supergravity solution of a gravitational plane wave with a constant, null five-form field strength constitutes a maximally supersymmetric background:

$$ds^2 = -2dx^+dx^- - \mu^2 x_I^2 (dx^+)^2 + dx_I{}^2, \qquad (1.1)$$
$$F_{+1234} = F_{+5678} = 2\mu.$$

The plane wave geometry (1.1) is obtained by taking the Penrose limit of the $AdS_5 \times S^5$ geometry.

The $AdS_5 \times S^5$ geometry got a prominent position due to the AdS/CFT duality [2–4] asserting that the type IIB superstring moving in the $AdS_5 \times S^5$ background is dual to the four dimensional $\mathcal{N} = 4$ super Yang-Mills theory. Though a remarkable success of this conjecture, a general proof is still out of reach since the string theory in this background is given by a highly nonlinear two dimensional field theory [5] and the duality relates the weak coupling regime of one theory to the strong coupling regime of the other theory. Since the plane wave geometry (1.1) is obtained through a limit of the $AdS_5 \times S^5$ geometry, this limit is particularly interesting by virtue of the AdS/CFT duality. It was realized in [6] that the type IIB string theory in the plane wave background (1.1) has a very simple description in terms of the dual supersymmetric Yang-Mills theory in a particular double scaling limit. Remarkably the duality turns out be perturbatively accessible from both sides of the correspondence, which so truly goes to a regime of interacting string theory. For reviews on this subject, see, for example, [7].

This kind of concrete realization of the duality is mainly due to the fact that the string theory in the Ramond-Ramond background (1.1) is exactly solvable [8, 9]. The plane wave superstring reduces to a free, massive two dimensional model once one goes to the light-cone gauge. It is therefore as straightforwardly quantized as the superstring in a flat spacetime background. It may thus be possible to get the complete spectrum of the plane wave superstring including D-branes too.

Since D-branes play a very crucial role in the understanding of string dualities, AdS/CFT duality, microscopic description of some black-hole entropy, phenomenological model building, etc. [10, 11], it is important to have a complete classification of D-branes. In a flat spacetime, the D-branes appear as the half BPS solitons of the type II string theories, preserving 16 supersymmetries and their transverse positions can be arbitrary so that they are usually the moduli of the BPS solitons. D-branes can also be described by the boundary states of closed strings [12, 13]. The symmetries that the boundary state preserves are thus generically the combination of the closed string symmetries that leave the boundary state invariant. In this scheme, a D-brane acts as a source of closed strings and such properties are guaranteed by the conformal symmetry of the worldsheet.

Though the properties of D-branes have been extensively studied over a decade, it is still a challenging problem to completely classify the D-branes in a general string background. Since the string propagation on the curved background (1.1) can be solved exactly by choosing light-cone gauge in the Green-Schwarz action [8, 9], we may have a systematic classification of D-branes in the type IIB plane wave background. Two of us with Cha showed in [14] that it is actually possible at least for longitudinal branes, i.e., extended along the light-cone directions. In this paper we will extend the previous work [14] to less supersymmetric configurations by introducing magnetic as well as electric fluxes, or constantly boosting D-branes. We will find a very rich spectrum of supersymmetric D-branes in the type IIB plane wave background. D-branes in the Ramond-Ramond background (1.1) have been studied in a number of papers [14]–[40] from different points of view. Branes in other plane wave backgrounds have also been studied [41]–[47].

This paper is organized as follows. In section 2 we review the D-brane classification in [14]. In section 3 we start with eq. (3.7) derived in [14], which is the most general worldsheet supercurrent for the open string dynamical supersymmetry applicable to D-branes with an electric flux F_{+I} and an angular momentum L_{IJ}, i.e., boosted D-branes.

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We find all possible configurations preserving some fraction of dynamical supersymmetries for the $D_{\pm}$-branes [16]–[23] and the oblique D-branes (OD-branes) [24 − 26, 14]. We will not only reproduce the known solutions but also find many new supersymmetric D-branes. In section 4 we consider D-branes in a magnetic flux background F_{IJ}, which was recently analyzed by Mattik [15] for maximally supersymmetric D-branes. We derive the most general worldsheet supercurrent (4.14) with the magnetic flux F_{IJ}. We find several supersymmetric D-branes, of course, with reproducing the maximally supersymmetric cases found by Mattik [15]. In section 5 we study D-branes in the most general background, say, with F_{+I}, L_{IJ} as well as F_{IJ}. We find there still exist some supersymmetric D-branes even in this case. In section 6 we briefly review our results obtained and discuss some related issues. Finally, in appendix A we list useful matrix relations which are used to find supersymmetric backgrounds for the oblique D-branes.

2. D-branes in a plane wave background

The Green-Schwarz light-cone action in the plane wave background (1.1) describes eight free massive bosons and fermions [8, 9]. In the light-cone gauge, $X^+ = \tau$, the action is given by

$$S = \frac{1}{2\pi\alpha' p^+} \int d\tau \int_0^{2\pi\alpha'|p^+|} d\sigma \left[\frac{1}{2}\partial_+ X_I \partial_- X_I - \frac{1}{2}\mu^2 X_I^2 - i\bar{S}(\rho^A \partial_A - \mu\Pi)S \right] \tag{2.1}$$

where $\partial_\pm = \partial_\tau \pm \partial_\sigma$. The equations of motion following from the action (2.1) take the form

$$\partial_+\partial_- X^I + \mu^2 X^I = 0, \tag{2.2}$$

$$\partial_+ S^1 - \mu\Pi S^2 = 0, \qquad \partial_- S^2 + \mu\Pi S^1 = 0. \tag{2.3}$$

The closed string action (2.1) has the global symmetry $SO(4) \times SO(4)' \times \mathbf{Z}_2$ which is the isometry in the plane wave background (1.1). The $\mathbf{Z}_2$ symmetry here interchanges simultaneously the two SO(4) directions [48][1]

$$\mathbf{Z}_2 : (x^1, x^2, x^3, x^4) \leftrightarrow (x^5, x^6, x^7, x^8). \tag{2.4}$$

In this paper we want to study supersymmetric D-branes in the plane wave background (1.1). One of doing this is, according to Polchinski [10, 11], to consider an open string attached on a Dp-brane. The open string theory is then defined by the action (2.1) with appropriate boundary conditions imposed on each end of the open string. So our interest is to find what boundary condition has to be imposed to preserve (dynamical) supersymmetries in the open string theory on the Dp-brane. Following the recipe in [14], we will present an efficient worldsheet formalism for the supersymmetric boundary conditions on the most general ground.

[1] This $\mathbf{Z}_2$ symmetry explains why the oblique D-brane, which is at 45^o angle in the two SO(4) directions, can exist in the plane wave background (1.1) and the spectrum of D-branes is symmetric under the $\mathbf{Z}_2$ involution (2.4).

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Since the boundary condition of fermionic coordinates is insensitive to the details of bosonic boundary conditions, we assume the following boundary condition at each end of the open string [49] without loss of generality:

$$(S^1 - \Omega S^2)|_{\partial\Sigma} = 0, \tag{2.5}$$

where Ω is a fermionic gluing matrix whose explicit form will be specified.

For D-branes with the flux F_{+I} and the angular momentum L_{IJ} only, the gluing matrix Ω is exactly the same as the trivial backgrounds and is simply given by the product of γ-matrices along the Neumann directions:[2]

$$\Omega = \prod_{r\in N} \gamma^r. \tag{2.6}$$

So, in this case,

$$\Omega^2 = \pm 1, \tag{2.7}$$

$$\gamma^r \Omega = -\Omega \gamma^r, \quad \forall r \in N, \tag{2.8}$$

$$\gamma^{r'} \Omega = \Omega \gamma^{r'}, \quad \forall r' \in D. \tag{2.9}$$

For D-branes with the flux F_{rs}, however, Ω has the following form [13, 15]:

$$\Omega = \widetilde{\Omega} \exp^{\frac{1}{4}\Theta_{rs}\gamma^{rs}}, \tag{2.10}$$

where $\widetilde{\Omega}$ is the gluing matrix of the type (2.6) for the Neumann directions without flux and the parameters Θ_{rs} depend on the flux F_{rs}. In this case, the nice properties, eqs. (2.7) and (2.8), no longer hold due to the additional exponential factor. However the property (2.9) is still true since the flux F_{rs} extends only along the Neumann directions. If $\Theta_{rs} \neq 0$, e.g., with rank 2, the gluing matrix Ω continuously interpolates among codimension 2 D-branes. When $\Theta_{rs} \to 0$ or π, we have to recover the case (2.6) [15].

As was shown in [14], the possible type of the D-branes with the gluing matrix Ω in eq. (2.6) can be characterized by the matrix Γ defined by

$$\Gamma \equiv \Pi\Omega\Pi\Omega. \tag{2.11}$$

It is easy to show that the matrix Γ satisfies the following relations:

$$\Pi\Omega\Pi\Omega = \Gamma = \Pi\Omega^T\Pi\Omega^T, \tag{2.12}$$

$$\Gamma\Gamma^T = 1, \qquad \Pi\Gamma\Pi\Gamma = 1. \tag{2.13}$$

The matrix Γ is either a symmetric or an antisymmetric matrix. In the case the matrix Γ is symmetric, i.e. $\Gamma^T = \Gamma$, it follows from (2.12) and (2.13) that

$$\Gamma^2 = 1, \qquad [\Pi, \Gamma] = 0 = [\Omega, \Gamma]. \tag{2.14}$$

[2] In this paper we will use the notation and the convention in [14]. Neumann (N) coordinates X^r are decomposed into oblique directions $X^{\hat{r}}$ and usual parallel directions $X^{\dot{r}}$: $r = (\hat{r}, \dot{r})$. Similarly, Dirichlet (D) coordinates $X^{r'}$ are also decomposed into oblique directions $X^{\hat{r}'}$ and usual parallel directions $X^{\dot{r}'}$: $r' = (\hat{r}', \dot{r}')$.

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D-brane type	Γ	Ω
$D_\pm$	± 1	$\Omega_{D_\pm}$
$OD3$	$\pm\gamma^{1256}$	$\frac{1}{2}(\gamma^1-\gamma^6)(\gamma^2\pm\gamma^5)$
$OD5$	$\pm\gamma^{1256}$	$\frac{1}{2}(\gamma^1-\gamma^6)(\gamma^2\mp\gamma^5)\gamma^{34}$ $\frac{1}{2}(\gamma^1-\gamma^6)(\gamma^2\mp\gamma^5)\gamma^{78}$ $\frac{1}{2}(\gamma^1-\gamma^6)(\gamma^2\pm\gamma^5)\gamma^{37}$
$OD7$	$\pm\gamma^{1256}$	$\frac{1}{2}(\gamma^1-\gamma^6)(\gamma^2\pm\gamma^5)\gamma^{3478}$
$OD_\pm 5$	$\pm\gamma$	$\frac{1}{4}(\gamma^1-\gamma^6)(\gamma^2\pm\gamma^5)(\gamma^3-\gamma^8)(\gamma^4+\gamma^7)$

Table 1: D-branes with $\Gamma^2 = 1$

On the other hand, in the case the matrix Γ is antisymmetric, i.e. $\Gamma^T = -\Gamma$,

$$\Gamma^2 = -1, \qquad \{\Pi, \Gamma\} = 0 = \{\Omega, \Gamma\}. \tag{2.15}$$

It was shown in [14] that the D-branes satisfying $\Gamma^2 = -1$ preserve no supersymmetry. This fact is not affected by introducing nontrivial backgrounds since the gluing matrix Ω is still the same as before and the matrix Γ then has imaginary eigenvalues. Thus we will consider only the D-branes satisfying $\Gamma^2 = 1$. Table 1 shows the possible D-branes with particular polarizations. Other D-branes with different polarizations can be obtained by the $SO(4) \times SO(4)'$ rotations.

$D_\pm$-branes [16]–[23] are a specific class satisfying $\Gamma = \pm 1$, which are denoted as $(+, -, m, n)$ with $m, n = 0, 1, \ldots, 4$ following the convention in [18]. D_--branes are of the type $|m - n| = 2$ while D_+-branes are of the type $|m - n| = 0, 4$. The oblique D-branes with $\Gamma^2 = 1$ can be summarized as follows:

$$ODp\text{-brane} : \Gamma = \pm\gamma^{i_1 i_2 i'_3 i'_4}, \quad (p = 3, 5, 7), \tag{2.16}$$

$$OD5\text{-brane} : \Gamma = \pm\gamma, \tag{2.17}$$

where $\gamma = \gamma^{12\ldots8}$ is the SO(8) chirality matrix. Eq. (2.14) requires that Γ should contain an even number of gamma matrices in both $\{\gamma^i,\ i = 1, \ldots, 4\}$ and $\{\gamma^{i'},\ i' = 5, \ldots, 8\}$.

3. Supersymmetric D-branes with F_{+I} and L_{IJ}

In a light-cone gauge, the 32 components of the supersymmetries for a closed string decompose into kinematical supercharges, Q_a^{+A}, and dynamical supercharges, $Q_{\dot a}^{-A}$. For a closed superstring in the plane wave background with the action (2.1), the conserved super-Nöther charges were identified by Metsaev [8]:

$$Q^{+1} = \frac{\sqrt{2p^+}}{2\pi\alpha' p^+} \int_0^{2\pi\alpha'|p^+|} d\sigma (\cos\mu\tau S^1 - \sin\mu\tau \Pi S^2), \tag{3.1}$$

$$Q^{+2} = \frac{\sqrt{2p^+}}{2\pi\alpha' p^+} \int_0^{2\pi\alpha'|p^+|} d\sigma (\cos\mu\tau S^2 + \sin\mu\tau \Pi S^1), \tag{3.2}$$

$$\sqrt{2p^+}Q^{-1} = \frac{1}{2\pi\alpha' p^+}\int_0^{2\pi\alpha'|p^+|} d\sigma\Big(\partial_- X^I\gamma^I S^1 - \mu X^I\gamma^I\Pi S^2\Big), \tag{3.3}$$

$$\sqrt{2p^+}Q^{-2} = \frac{1}{2\pi\alpha' p^+}\int_0^{2\pi\alpha'|p^+|} d\sigma\Big(\partial_+ X^I\gamma^I S^2 + \mu X^I\gamma^I\Pi S^1\Big). \tag{3.4}$$

The kinematical supersymmetry is, in general, related to a shift of spinor fields and thus generated by spinor zero modes. So the kinematical supersymmetry is insensitive to the details of backgrounds, i.e., fluxes and boosting,[3] and it has to be fixed by the boundary condition (2.5). Since we are interested in the open string supersymmetry surviving non-trivial backgrounds, we will focus only on the dynamical supersymmetry. The dynamical supercharge preserved by an open string on a D-brane is given by a combination of closed string supercharges Q^{-A} compatible with the open string boundary conditions. Due to the boundary condition (2.5), it turns out that the conserved dynamical supercharge is given by (a subset of)

$$q^- = Q^{-1} - \Omega Q^{-2}. \tag{3.5}$$

In this section we will first show how D-branes can preserve dynamical supersymmetries by turning on the flux F_{+I} or the angular momentum L_{IJ}. It is easy to derive the conservation law [14] for the dynamical supersymmetry in eq. (3.5) using the equations of motion, eqs. (2.2) and (2.3):

$$\frac{\partial q^-_\tau}{\partial\tau} + \frac{\partial q^-_\sigma}{\partial\sigma} = 0, \tag{3.6}$$

where

$$\begin{aligned} q^-_\sigma = \sqrt{\frac{1}{2p^+}}\Big(&(\partial_\tau X^r\gamma^r - \partial_\sigma X^{r'}\gamma^{r'})(S^1 - \Omega S^2) \\ &+(\partial_\tau X^{r'}\gamma^{r'} - \partial_\sigma X^r\gamma^r)(S^1 + \Omega S^2) \\ &+\mu X^r\gamma^r\Omega\Pi(S^1 + \Gamma\Omega S^2) - \mu X^{r'}\gamma^{r'}\Omega\Pi(S^1 - \Gamma\Omega S^2)\Big). \end{aligned} \tag{3.7}$$

In the course of derivation, we used the relations, (2.8) and (2.9). However, we didn't assume anything about bosonic as well as fermionic boundary conditions.

In order for the supercharge q^- to be conserved, the current q^-_σ in eq. (3.7) has to vanish at the boundary of worldsheet, $\partial\Sigma$. Now we assume the fermionic boundary condition (2.5), but it does not loose any generality since (the form of) the boundary condition (2.5) does not depend on the details of backgrounds. Then we will find what boundary conditions for bosonic coordinates X^I have to be imposed to get the vanishing current at the boundary, i.e., $q^-_\sigma|_{\partial\Sigma} = 0$. Since the details of the bosonic boundary condition, however, depend on the type of D-brane, we will discuss $D_\pm$-branes and OD-branes, separately.

[3]But the explicit form of the spinor zero modes themselves is sensitive to the type of D-brane and the background gauge condensates [14, 15].

3.1 D_--branes

First we consider the dynamical supersymmetry of D_--branes, where $\Gamma = -1$. In this case, the current q_σ^- in eq. (3.7) at the boundary reduces to

$$q_\sigma^-|_{\partial\Sigma} = \sqrt{\frac{2}{p^+}}\Big(\partial_\tau X^{r'}\gamma^{r'} - \partial_\sigma X^r\gamma^r - \mu X^{r'}\gamma^{r'}\Omega\Pi\Big)S^1|_{\partial\Sigma}. \tag{3.8}$$

We want to find what conditions are needed for bosonic coordinates X^I in order for the current (3.8) to vanish at the boundary. Of course, the trivial case is

$$\partial_\tau X^{r'} = \partial_\sigma X^r = X^{r'} = 0, \quad \forall r' \in D,\, \forall r \in N, \tag{3.9}$$

and this configuration preserves maximal supersymmetry. But, as we will discuss, there are many other configurations with the vanishing current which so preserve some amount of supersymmetries.

It is useful to notice that the matrix $\Omega\Pi$ for the D_--branes takes the following form:

$$\Omega\Pi = \pm\gamma^{I_1I_2} \;\text{ or }\; \pm\gamma\gamma^{I_1I_2}, \tag{3.10}$$

where

$$D_-3\text{-brane}:\; (I_1, I_2) \in D, \tag{3.11}$$

$$D_-5\text{-brane}:\; I_1 \in N,\;\; I_2 \in D, \tag{3.12}$$

$$D_-7\text{-brane}:\; (I_1, I_2) \in N. \tag{3.13}$$

3.1.1 $\partial_\sigma X^r \neq 0$ case

If we consider the case $X^{r'}|_{\partial\Sigma} \equiv x_0^{r'} \neq 0$ for some $r' \in D$, our problem is reduced to that finding a matrix satisfying

$$\gamma^{rr'}\Omega\Pi S^1 = \pm S^1. \tag{3.14}$$

A necessary condition is that $(\gamma^{rr'}\Omega\Pi)^2 = 1$. Therefore the matrix $\gamma^{rr'}\Omega\Pi$ has to take the form

$$\gamma^{rr'}\Omega\Pi = \pm\gamma^{I_1\ldots I_n}, \quad n = 0, 4, 8. \tag{3.15}$$

If a matrix exists satisfying eq. (3.14), the current at the boundary can vanish with the following modified Neumann boundary condition:

$$\partial_\sigma X^r - \mu x_0^{r'} = 0. \tag{3.16}$$

This kind of boundary condition can be easily achieved by introducing a boundary coupling with the worldvolume gauge field $A_+ = -F_{+r}X^r$:

$$S_B = -\frac{1}{2\pi\alpha' p^+}\int_{\partial\Sigma} d\tau A_\mu(X)\frac{\partial X^\mu}{\partial\tau} = \frac{F_{+r}}{2\pi\alpha' p^+}\int_{\partial\Sigma} d\tau X^r. \tag{3.17}$$

When the spinor S^A satisfies (3.14) together with the Neumann boundary condition (3.16), the dynamical supersymmetries given by $\frac{1}{2}(1 \pm \gamma^{rr'}\Omega\Pi)q^-$ are preserved.

Going with eq. (3.10) into eq. (3.15), it is easy to see that the $n = 0$ and 8 cases are possible only for D5-branes: $(+, -, 3, 1)$ and $(+, -, 1, 3)$ which preserve maximal supersymmetry as was shown in [18, 36, 14]. For example, let us take a $(+, -, 3, 1)$-brane extended along $(+, -, 1, 2, 3, 5)$ directions, say, $N = (1, 2, 3, 5)$ and $D = (4, 6, 7, 8)$ and thus $\Omega\Pi = -\gamma^{45}$. In this case we need the flux $F_{+5} = \mu x_0^4$ only.

Using eq. (3.10), it is obvious that the $n = 4$ case is possible for all D_--branes. Four dynamical supersymmetries are preserved in this case. We will not give any detail since it should be really simple. Instead let us give you an example: Consider $(+, -, 2, 0)$-brane where $\Omega = \gamma^{12}$ and $\Omega\Pi = -\gamma^{34}$. If $X^5|_{\partial\Sigma} = x_0^5 \neq 0$ and the spinor satisfies $(1 \pm \gamma^{1345})S^1 = 0$ at the boundary, the half of dynamical supercharges are preserved with the boundary condition $\partial_\sigma X^1 \mp \mu x_0^5 = 0$.

We can get less supersymmetric configurations by considering more general backgrounds. For example, let us consider two matrices M_1 and M_2 satisfying eq. (3.14). In order for the spinor S^1 to simultaneously satisfy the condition (3.14) for this background, the product of M_1 and M_2, $M_3 = M_1 M_2$, should again be of the form (3.15). In a pedantic notation,

$$M_1 S^1 = \pm S^1 \text{ and } M_2 S^1 = \pm S^1 \;\Rightarrow\; M_3 = M_1 M_2 = \pm\gamma^{I_1 \dots I_n}, \quad n = 0, 4, 8. \tag{3.18}$$

If M_3 is of the form with $n = 0, 8$, the supersymmetry is not further broken. But, the dynamical supersymmetry is further broken by half in the case of $n = 4$.

What is the least supersymmetric configuration which can be realized by turning on constant fluxes F_{+I} ? Since $M_3 = M_1 M_2$ should be of the form in eq. (3.18), we can see from eqs. (3.11)–(3.13) that $D3$- and $D7$-branes can have only two independent projections - 2 dynamical supersymmetries. This can be easily understood by noting that the $D3$ ($D7$)-brane has only two Neumann (Dirichlet) directions. For the D5-brane discussed above, for example, we can have $M_1 = \gamma^{1845}$, $M_2 = \gamma^{2745}$ and $M_3 = \gamma^{3645}$, but $M_1 M_2 M_3 = -\gamma$, so M_1, M_2 and M_3 cannot be simultaneously independent in the space of positive chirality spinors. Therefore the D5-brane also preserves at least 2 dynamical supersymmetries.

3.1.2 $\partial_\tau X^{r'} \neq 0$ case

If we consider the case $X^{s'}|_{\partial\Sigma} \equiv v^{s'} \neq 0$ for some $s' \in D$, we need a modified Dirichlet boundary condition:

$$\partial_\tau X^{r'} - \mu v^{s'} = 0. \tag{3.19}$$

This kind of boundary condition can be achieved by boosting a D-brane with constant velocity $v^{s'}$ in a transverse direction. This means we are considering the following transformation

$$X^{r'} \to X^{r'} - \mu v^{s'} \tau \tag{3.20}$$

where the light-cone gauge $X^+ = \tau$ is used. With the boundary condition (3.19), the supersymmetric condition is reduced to that finding a matrix satisfying

$$\gamma^{r's'}\Omega\Pi S^1 = \pm S^1. \tag{3.21}$$

Therefore the matrix $\gamma^{r's'}\Omega\Pi$ has to take the form

$$\gamma^{r's'}\Omega\Pi = \pm\gamma^{I_1\ldots I_n}, \quad n = 0, 4, 8. \tag{3.22}$$

Note that $\gamma^{r's'}$ is a SO(2) spinor rotation in the transverse rotational symmetry SO$(4-m) \times$ SO$(4-n)$ for a $(+,-,m,n)$-brane.

Going with eq. (3.10) into eq. (3.22), it is easy to see that the $n=0$ and 8 cases are possible only for D3-branes: $(+,-,2,0)$ and $(+,-,0,2)$ which preserve maximal supersymmetry as was shown in [36]. This is the case that $\gamma^{r's'} \in$ SO(2) in the transverse rotation symmetry SO(4) $\times$ SO(2). However, the $n=4$ case is possible for all D_--branes in which case four dynamical supersymmetries are preserved. These branes are rotating in the $X^{r'}$-$X^{s'}$ plane and correspond to the giant gravitons.

We can get less supersymmetric configurations by considering more boostings. What is the least supersymmetric configuration which can be realized by boosting a D-brane ? If we consider two boosts simultaneously, the product of M_1 and M_2, $M_3 = M_1M_2$, should be of the form in eq. (3.22) where the matrices M_1 and M_2 satisfy eq. (3.21). Then we can see from eqs. (3.11)–(3.13) that $D5$- and $D7$-branes can have only one supersymmetric rotation - 4 dynamical supersymmetries. This can be easily understood by noting that the SO(3) (SO(2)) rotation for the $D5$ ($D7$)-brane is rank 1. For the D3-brane, however, we can have two simultaneous rotations in the transverse SO(4) directions - 2 dynamical supersymmetries since SO(4) is rank 2. The simultaneous SO(2) rotation of the D3-brane does not further break supersymmetry as the reason discussed above.

3.1.3 general case

Now we consider general cases with $\partial_\tau X^{r'} \neq 0$ and $\partial_\sigma X^r \neq 0$. In this case we have two kinds of matrix from the conditions (3.14) and (3.21). One is of the form $M^F = \gamma^{rr'}\Omega\Pi$ and the other is $M^L = \gamma^{s't'}\Omega\Pi$. To preserve the dynamical supersymmetry, the following condition is further required:

$$M^F M^L = \pm\gamma^{rr's't'}. \tag{3.23}$$

Thus we need at least three Dirichlet directions. Note that we can simply add the maximally supersymmetric configuration in the previous cases not affecting the resulting supersymmetry only if the condition (3.23) is satisfied. So we will discuss supersymmetric configurations up to the maximally supersymmetric background in 3.1.1 and 3.1.2.

The condition (3.23) says that this case can preserve at most 4 dynamical supersymmetries. It also says that the D7-brane cannot preserve any dynamical supersymmetry in this case. Noting that $M^L = \gamma^{rr's't'}$ for the D5-brane, the background with one flux and one rotation can preserve 2 dynamical supersymmtry as the least supersymmetric configuration. For the D3-brane, first note that $M^F = \gamma^{rr's't'}$, $M^L = \gamma^{r's't'u'}$ and so we can have only two independent projections satisfying the condition (3.23). For example, for the D3-brane discussed in 3.1.1, $M_1^F = \gamma^{1345}$, $M_2^F = \gamma^{2346}$ and $M_1^L = \gamma^{3478}$. Since $M_1^F M_2^F M_1^L = \gamma$, the dynamical supersymmetry is reduced only by 1/4.

3.2 D_+-branes

Next we consider the dynamical supersymmetry of D_+-branes, where $\Gamma = +1$. In this case, the current q_σ^- in eq. (3.7) at the boundary reduces to

$$q_\sigma^-|_{\partial\Sigma} = \sqrt{\frac{2}{p^+}}\Big(\partial_\tau X^{r'}\gamma^{r'} - \partial_\sigma X^r\gamma^r + \mu X^r\gamma^r\Omega\Pi\Big)S^1|_{\partial\Sigma}. \tag{3.24}$$

A crucial difference from the D_--branes is that the term proportional to μ is now involved with Neumann coordinates, which are in general nonvanishing and τ-dependent at the boundary. So we can realize a supersymmetric configuration neither by turning on a constant flux nor by boosting the D-brane unlike as D_--branes.

Nevertheless, as was found in [14], the dynamical supersymmetry can be preserved by introducing a boundary coupling with the worldvolume gauge field:

$$S_B = -\frac{1}{2\pi\alpha' p^+}\int_{\partial\Sigma} d\tau A_\mu(X)\frac{\partial X^\mu}{\partial\tau} = -\frac{1}{2\pi\alpha' p^+}\int_{\partial\Sigma} d\tau A_+(X), \tag{3.25}$$

where the flux F_{+I} is not constant but linearly depends on the Neumann coordinates. That is, the gauge field $A_+(X)$ is given by

$$A_+(X) = \pm\frac{\mu}{2}\Bigg(\sum_{r_1\in N_1} X^{r_1}X^{r_1} - \sum_{r_2\in N_2} X^{r_2}X^{r_2}\Bigg), \tag{3.26}$$

where N_1 denotes Neumann coordinates in the first SO(4) directions and N_2 does those in the second SO(4) directions. The Neumann boundary condition is then modified as follows

$$\Big(\partial_\sigma X^{r_1} \pm \mu X^{r_1}\Big)_{\partial\Sigma} = 0 = \Big(\partial_\sigma X^{r_2} \mp \mu X^{r_2}\Big)_{\partial\Sigma}. \tag{3.27}$$

The dynamical supersymmetry of D_+-branes can be preserved basically due to the fact that $(\Omega\Pi)^2 = 1$ so that there are always solutions satisfying $\Omega\Pi S^1 = \pm S^1$. In particular, the $(+,-,4,0)$- and $(+,-,0,4)$-brane preserve the maximal supersymmetry since $\Omega\Pi = 1$ and γ, respectively, for these branes [18, 21, 23]. One may ask whether or not less supersymmetric configurations can be constructed. Looking into the structure of the current in eq. (3.24), it seems to be impossible.

3.3 OD-branes

According to the gluing matrix Ω in table 1, we will define diagonal coordinates

$$X^{\hat r} = \frac{1}{\sqrt{2}}(X^r \pm X^{r'}), \qquad X^{\hat r'} = \frac{1}{\sqrt{2}}(X^{r'} \mp X^r) \tag{3.28}$$

with the index notation explained in footnote 2. For an OD5-brane described by $\Omega = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 - \gamma^5)\gamma^{34}$, for example, we have

$$\text{Neumann}: X^{\hat 1} = \frac{1}{\sqrt{2}}(X^1 - X^6),\ X^{\hat 2} = \frac{1}{\sqrt{2}}(X^2 - X^5),\ X^{\dot 3} = X^3,\ X^{\dot 4} = X^4,$$

$$\text{Dirichlet}: X^{\hat 5'} = \frac{1}{\sqrt{2}}(X^5 + X^2),\ X^{\hat 6'} = \frac{1}{\sqrt{2}}(X^6 + X^1),\ X^{\dot 7'} = X^7,\ X^{\dot 8'} = X^8.$$

To discuss the supersymmetry of OD-branes, it is useful to decompose the spinors $S^A(\tau,\sigma)$ into the eigenspinors of Γ by defining

$$S^A_\pm(\tau,\sigma) = P_\pm S^A(\tau,\sigma), \tag{3.29}$$

where

$$P_\pm = \frac{1}{2}(1\pm\Gamma). \tag{3.30}$$

It follows from eq. (2.14) that the equations of motion, eq. (2.3), are completely separated into two independent equations of motion for the spinors $S^A_\pm(\tau,\sigma)$

$$\partial_+ S^1_+ - \mu\Pi S^2_+ = 0, \qquad \partial_- S^2_+ + \mu\Pi S^1_+ = 0, \tag{3.31}$$
$$\partial_+ S^1_- - \mu\Pi S^2_- = 0, \qquad \partial_- S^2_- + \mu\Pi S^1_- = 0 \tag{3.32}$$

and the boundary condition, eq. (2.5), can be separately imposed for the spinors $S^A_\pm(\tau,\sigma)$

$$(S^1_+ - \Omega S^2_+)|_{\partial\Sigma} = 0, \tag{3.33}$$
$$(S^1_- - \Omega S^2_-)|_{\partial\Sigma} = 0. \tag{3.34}$$

It can be shown [14] that the spinor $S^A_+(\tau,\sigma)$ then has a D_+-like mode expansion while $S^A_-(\tau,\sigma)$ does a D_--like mode expansion since

$$\Gamma S^A_\pm(\tau,\sigma) = \pm S^A_\pm(\tau,\sigma). \tag{3.35}$$

Since the condition for $q^-_\sigma|_{\partial\Sigma}$ in eq. (3.7) to vanish depends on the eigenvalue of the matrix Γ as was reasoned above, we introduce projected supercharges defined by

$$q^-_\pm \equiv P_\pm(Q^{-1} - \Omega Q^{-2}). \tag{3.36}$$

It is easy to get the value of the current $q^-_{\pm\sigma}$ at the boundary:

$$\begin{aligned} q^-_{+\sigma}\Big|_{\partial\Sigma} = \sqrt{\frac{2}{p^+}}\Big(& (\partial_\tau X^{\check r'}\gamma^{\check r'} - \partial_\sigma X^{\check r}\gamma^{\check r} + \mu X^{\check r}\gamma^{\check r}\Omega\Pi)S^1_+ \\ & +(\partial_\tau X^{\hat r'}\gamma^{\hat r'} - \partial_\sigma X^{\hat r}\gamma^{\hat r} - \mu X^{\hat r'}\gamma^{\hat r'}\Omega\Pi)S^1_-\Big)_{\partial\Sigma} \end{aligned} \tag{3.37}$$

and

$$\begin{aligned} q^-_{-\sigma}\Big|_{\partial\Sigma} = \sqrt{\frac{2}{p^+}}\Big(& (\partial_\tau X^{\hat r'}\gamma^{\hat r'} - \partial_\sigma X^{\hat r}\gamma^{\hat r} + \mu X^{\hat r}\gamma^{\hat r}\Omega\Pi)S^1_+ \\ & +(\partial_\tau X^{\check r'}\gamma^{\check r'} - \partial_\sigma X^{\check r}\gamma^{\check r} - \mu X^{\check r'}\gamma^{\check r'}\Omega\Pi)S^1_-\Big)_{\partial\Sigma}. \end{aligned} \tag{3.38}$$

Note that the dynamical supersymmetry, q^-_+ and q^-_-, cannot simultaneously be preserved since each set of boundary conditions cannot simultaneously be compatible with each other. So we will separately consider the supercharges $q^-_\pm$.

We see that the $+$ component of the spinor S^1 in eqs. (3.37)–(3.38) gives D_+-like supercharge, while the $-$ component gives D_--like supercharge. As was shown in [14], the D_+-like supercharge can be preserved by turning on a boundary coupling with the gauge field $A_+(X)$ like eq. (3.26). One can easily understand the results by looking into the matrix relations in appendix A. Especially, the $OD3$-brane preserves 4 dynamical supersymmetries q_+^- [24, 25] since $X^{\dot{r}} = 0$ by definition while it can do only 2 dynamical supersymmetries q_-^- by turning on a boundary coupling with the gauge field $A_+(X)$ of the type (3.26). The $OD5$-branes with $\Omega = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 - \gamma^5)\gamma^{34}$ and $\Omega = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 - \gamma^5)\gamma^{78}$ also preserve $q_\pm^-$ without any further projection since they satisfy eq. (A.3) and eq. (A.5), respectively.

We go over to the $-$ component of the spinor S^1 which gives D_--like supercharge. We are now interested in the situation $X^{r'} \neq 0$ for some $r' \in D$. One has to remember that we already introduced one or two projection operators to preserve the D_+-like supercharge, so that the introduction of nontrivial backgrounds for the D_--like supercharge may further break the supersymmetry. It could be helpful to have an analogue of eq. (3.10) for the OD-branes. In appendix A we list the useful matrix relations for those in table 1. The matrix relations show a quite similar property to eq. (3.10) so that we can apply the same strategy as the D_--branes. For this, we will often use the simple fact, for example,

$$(\gamma^I - \gamma^J)(\gamma^I + \gamma^J) = 2\gamma^{IJ}. \tag{3.39}$$

We will not repeat how to modify the boundary conditions for the OD-branes by turning on a flux or boosting a D-brane since it is essentially the same as the D_--branes.

Let us first discuss the $OD_\pm 5$-branes since they are special compared to other OD-branes. As was shown in [14], the OD_+5-brane preserves no dynamical supersymmetry. For the OD_-5-brane, however, the current $q^-_{+\sigma}$ at the boundary (q_-^- identically vanishes) is given by

$$q^-_{+\sigma}\Big|_{\partial\Sigma} = \sqrt{\frac{2}{p^+}}(\partial_\tau X^{\hat{r}'}\gamma^{\hat{r}'} - \partial_\sigma X^{\hat{r}}\gamma^{\hat{r}} - \mu X^{\hat{r}'}\gamma^{\hat{r}'}\Omega\Pi)S^1|_{\partial\Sigma}. \tag{3.40}$$

So the trivial boundary condition $\partial_\tau X^{\hat{r}'} = \partial_\sigma X^{\hat{r}} = X^{\hat{r}'} = 0$ preserves the maximal supersymmetry [24, 25]. Now our question is whether or not some dynamical supersymmetry can be preserved by introducing a constant flux or a boosting. The answer is no since Ω contains too many (4) oblique Neumann directions and so the vanishing condition in eq. (3.40) can also be involved with the product of 2 or 6 gamma matrices. In the following we will thus discuss the other OD-branes only.

3.3.1 $\partial_\sigma X^r \neq 0$ case

In this case the problem is to find a matrix satisfying

$$q_+^- : \ \gamma^{\hat{r}\hat{r}'}\Omega\Pi S_-^1 = \pm S_-^1 \tag{3.41}$$

or

$$q_-^- : \ \gamma^{\dot{r}\dot{r}'}\Omega\Pi S_-^1 = \pm S_-^1, \tag{3.42}$$

where the $OD3$-brane can preserve q_-^- with trivial Dirichlet boundary condition $X^{\dot{r}'} = 0$ only since $X^{\dot{r}} = 0$ by definition while the $OD7$-brane can preserve q_-^- with trivial Neumann boundary condition $\partial_\sigma X^{\dot{r}} = 0$ since $X^{\dot{r}'} = 0$. Otherwise we are implicitly assuming $X^{r'} \neq 0$

for the related Dirichlet coordinates. Of course, the supersymmetry $q_{\pm}^{-}$ can be preserved with the trivial boundary condition when $X^{r'} = 0$, which is not the case of our interest. For the gamma matrices in eq. (3.41), it is convenient to distinguish the following two cases

$$\gamma^{\hat{r}'} = \pm\Pi\gamma^{\hat{r}}\Pi, \tag{3.43}$$

$$\gamma^{\hat{r}'} \neq \pm\Pi\gamma^{\hat{r}}\Pi, \tag{3.44}$$

since their supersymmetry will be different in general.

From eqs. (A.4) and (A.6), we see that the corresponding $OD5$-branes preserve 4 dynamical supersymmetries q_{+}^{-} for the case (3.43) while no supersymmetry for the case (3.44). For example, the $OD5$-brane with $\Omega = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 - \gamma^5)\gamma^{34}$ has the value $\gamma^{\hat{r}\hat{r}'}\Omega\Pi = -\gamma^{1634}\gamma^{1234} = \gamma^{26}$ for the case (3.44). The other OD-branes preserve 2 dynamical supersymmetries q_{+}^{-} for the case (3.43) since we meet again the same projection operators as those in eqs. (A.1), (A.7) and (A.9). However the case (3.44) cannot preserve the supersymmetry q_{+}^{-} except the $OD3$-brane since the projection operators for the D_{-}-like supercharge are not compatible with those for the D_{+}-like supercharge. For example, the $OD5$-brane with $\Omega = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 + \gamma^5)\gamma^{37}$ has a value $\gamma^{\hat{r}\hat{r}'}\Omega\Pi = \gamma^{2537}\gamma^{1234} = \gamma^{1457}$ for the case (3.44) whose product with the matrices in eq. (A.7) becomes γ^{12} or γ^{56}. The $OD3$-brane did not yet use the matrices in eq. (A.1) for the D_{+}-like supercharge, so it preserves 2 dynamical supersymmetries q_{+}^{-} even for the case (3.44). On the other hand, we see that all the $OD5$-branes preserve 2 dynamical supersymmetries q_{-}^{-} since totally two independent projections are needed.

3.3.2 $\partial_\tau X^{r'} \neq 0$ case

In this case we have a condition

$$q_{+}^{-}: \ \gamma^{\hat{r}'\hat{s}'}\Omega\Pi S_{-}^{1} = \pm S_{-}^{1} \tag{3.45}$$

or

$$q_{-}^{-}: \ \gamma^{\dot{r}'\dot{s}'}\Omega\Pi S_{-}^{1} = \pm S_{-}^{1}, \tag{3.46}$$

where the $OD7$-brane does not belong to the case (3.46) since $X^{\dot{r}'} = 0$ by definition.

Noting that $\gamma^{\hat{r}'\hat{s}'}\gamma^{\hat{r}\hat{s}} = \pm\gamma^{1256}$ for the table 1 and following the similar reasoning to 3.3.1, we immediately see that only the $OD3$-brane preserves 2 dynamical supersymmetries q_{+}^{-}. Using the relations in appendix A, we also easily see that the $OD3$-brane preserves 2 dynamical supersymmetries q_{-}^{-} since no further projection is needed and the $OD5$-branes also do 2 dynamical supersymmetries except the $OD5$-brane with $\Omega = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 - \gamma^5)\gamma^{37}$ which preserves no supersymmetry q_{-}^{-} unless $X^{\dot{r}'} = 0$.

3.3.3 general case

Finally we consider the general case with $\partial_\tau X^{r'} \neq 0$ and $\partial_\sigma X^{r} \neq 0$. Since the conditions (3.41) and (3.45) or (3.42) and (3.46) have to be simultaneously satisfied, we have an additional condition as in 3.1.3 coming from the product

$$q_{+}^{-}: \ \gamma^{\hat{r}\hat{r}'}\Omega\Pi\gamma^{\hat{s}'\hat{t}'}\Omega\Pi = \gamma^{\hat{r}\hat{s}\hat{t}\hat{r}'}\Gamma, \tag{3.47}$$

$$q_{-}^{-}: \ \gamma^{\dot{r}\dot{r}'}\Omega\Pi\gamma^{\dot{s}'\dot{t}'}\Omega\Pi = \pm\gamma^{\dot{r}\dot{r}'\dot{s}'\dot{t}'}\Gamma, \tag{3.48}$$

where eq. (3.43) has been used in eq. (3.47). The $OD_{\pm}5$-brane and the $OD3$-brane only can satisfy eq. (3.47) and eq. (3.48), respectively. As was discussed in eq. (3.40), the $OD_{\pm}5$-brane cannot preserve the supersymmetry q_{+}^{-} in this case. On the other hand, since the term, $\partial_\sigma X^{\dot{r}}$, for the $OD3$-brane is absent in eq. (3.38), the $OD3$-brane does not belong to the present consideration but does to the previous case 3.3.2. Thus any dynamical supersymmetry of OD-branes is not preserved under the general background.

4. Supersymmetric D-branes with F_{IJ}

In this section we will study supersymmetric boundary conditions to preserve dynamical supersymmetries after turning on the flux F_{IJ} [15]. As we mentioned, the gluing matrix in this case is given by eq. (2.10). We can also derive the conservation law for the dynamical supersymmetry in eq. (3.5) with the gluing matrix Ω in eq. (2.10)

$$\frac{\partial q_\tau^-}{\partial \tau} + \frac{\partial q_\sigma^-}{\partial \sigma} = 0. \tag{4.1}$$

The current q_σ^- at the boundary is reduced to

$$q_\sigma^-|_{\partial\Sigma} = \sqrt{\frac{2}{p^+}}\Big[\partial_\tau X^{r'}\gamma^{r'}S^1 - \frac{1}{2}\Big(\partial_\sigma X^r - \Big(\frac{1-N}{1+N}\Big)^{rs}\partial_\tau X^s\Big)(\delta^{rt}+N^{rt})\gamma^t S^1 + \frac{\mu}{2}X^r\gamma^s(N^{rs}+\delta^{rs}\Gamma_B)\Omega\Pi S^1 - \frac{\mu}{2}X^{r'}\gamma^{r'}(1-\Gamma_B)\Omega\Pi S^1\Big], \tag{4.2}$$

where we defined

$$\Omega\gamma^r\Omega^T = -N^{rs}\gamma^s, \tag{4.3}$$
$$\Gamma_B = \Pi\Omega^T\Pi\Omega^T. \tag{4.4}$$

In eq. (4.2), we already used the fermionic boundary condition (2.5) and the relation (2.9). Note that $N^{rs} = \delta^{rs}$ when $F_{IJ} = 0$ and then we recover eq. (3.7).

Here we have taken a different recipe from Mattik's [15]. Indeed we only assumed the fermionic boundary condition (2.5), whose form is independent of the detail of backgrounds, to get the result (4.2). The relations (2.9) and (4.3) are the direct consequences (Baker-Campbell-Hausdorff formula) of the prescribed form of the gluing matrix Ω in eq. (2.10). We will now find most general boundary conditions which give rise to the vanishing current at the boundary, i.e., $q_\sigma^-|_{\partial\Sigma} = 0$.

It is convenient to divide the Neumann directions into two groups: $r = (a, i)$ where a, b, c denote the directions without magnetic flux and i, j, k denote those with magnetic flux. We also introduce a sign flip operation $\pi : X^{1,2,3,4} \mapsto -X^{1,2,3,4}$. In this notation, eq. (4.3) can be solved as follows:

$$\exp^{\frac{1}{4}\Theta_{jk}\gamma^{jk}}\,\gamma^i\,\exp^{-\frac{1}{4}\Theta_{jk}\gamma^{jk}} = -N^{ij}\gamma^j, \tag{4.5}$$
$$N^{ab} = \delta^{ab}. \tag{4.6}$$

Also the matrix Γ_B can be rewritten as follows

$$\Gamma_B = \widetilde{\Gamma}\exp^{-\frac{1}{4}\pi(\Theta_{jk})\gamma^{jk}}\exp^{-\frac{1}{4}\Theta_{jk}\gamma^{jk}}, \tag{4.7}$$

where $\widetilde{\Gamma} = \Pi\widetilde{\Omega}\Pi\widetilde{\Omega}$.

Using the above results, eq. (4.2) can be separably written into two parts:

$$q_\sigma^-|_{\partial\Sigma} = \sqrt{\frac{2}{p^+}}\Big[\Big(\partial_\tau X^{r'}\gamma^{r'} - \frac{\mu}{2}X^{r'}\gamma^{r'}(1-\Gamma_B)\Omega\Pi - \partial_\sigma X^a\gamma^a + \frac{\mu}{2}X^a\gamma^a(1+\Gamma_B)\Omega\Pi\Big)S^1$$
$$-\frac{1}{2}\Big(\partial_\sigma X^i + \mathcal{F}^{ij}\partial_\tau X^j\Big)(\delta^{ik}+N^{ik})\gamma^k S^1 + \frac{\mu}{2}X^i\gamma^j(N^{ij}+\delta^{ij}\Gamma_B)\Omega\Pi S^1\Big], \qquad (4.8)$$

where we defined

$$\mathcal{F}^{ij} = -\Big(\frac{1-N}{1+N}\Big)^{ij}. \qquad (4.9)$$

Looking into the terms in eq. (4.8), we see that the most pertinacious term is the last one, which is related to the Neumann coordinates and cannot be cancelled with other terms due to its peculiar form. So we have to demand [15] that

$$N^{ij}\gamma^j + \gamma^i\Gamma_B = 0. \qquad (4.10)$$

The above equation can be satisfied if and only if

$$\widetilde{\Gamma} = \Pi\widetilde{\Omega}\Pi\widetilde{\Omega} = 1, \qquad (4.11)$$
$$\gamma^i \exp^{-\frac{1}{4}\pi(\Theta_{jk})\gamma^{jk}} = \exp^{\frac{1}{4}\Theta_{jk}\gamma^{jk}}\gamma^i, \quad \forall i. \qquad (4.12)$$

The condition (4.12) is equivalent to

$$\pi(\Theta_{jk}) = \Theta_{jk} \;\text{ and }\; \mathrm{rank}(\Theta_{jk}) = 2. \qquad (4.13)$$

We see that the coordinates X^i in the limit $\Theta_{jk} = 0$ where $N^{ij} = -\delta^{ij}$ satisfy the usual Dirichlet boundary condition $\partial_\tau X^i = 0$ while in the limit $\Theta_{jk} = \pi$ where $N^{ij} = \delta^{ij}$ they satisfy the usual Neumann boundary condition $\partial_\sigma X^i = 0$. The conditions (4.11) and (4.13) thus say that we have to start from a D_+-brane when $\Theta_{jk} = 0$ and the magnetic flux should be extended along only two directions in $X^{1,2,3,4}$ or $X^{5,6,7,8}$ to have a D-brane to preserve the dynamical supersymmetry. So the D-brane with magnetic flux is continuously interpolating from a D_+-brane to a D_--brane with, in general, different amount of supersymmetries at the endpoints. We will see that the dynamical supersymmetry can be preserved by the same amount as D_+-branes only if the condition (4.10) is satisfied. So the maximally supersymmetric cases are $\widetilde{\Omega} = 1, \Pi, \gamma\Pi$ which correspond to $(+,-,0,0)$, $(+,-,4,0)$, $(+,-,0,4)$ branes when $\Theta_{jk} = 0$. These are exactly the cases found by Mattik [15].

Under the condition (4.10), the current in eq. (4.8) is reduced to

$$q_\sigma^-|_{\partial\Sigma} = \sqrt{\frac{2}{p^+}}\Big[\partial_\tau X^{r'}\gamma^{r'}S^1 - \partial_\sigma X^a\gamma^a S^1 + \mu X^a \cos\frac{\Theta_{jk}}{2}\gamma^a\widetilde{\Omega}\Pi S^1$$
$$-\mu X^{r'}\sin\frac{\Theta_{jk}}{2}\gamma^{r'}\gamma^{jk}\widetilde{\Omega}\Pi S^1 - \frac{1}{2}\Big(\partial_\sigma X^i + \mathcal{F}^{ij}\partial_\tau X^j\Big)(\delta^{ik}+N^{ik})\gamma^k S^1\Big]. \qquad (4.14)$$

Now it is easy to find bosonic boundary conditions to preserve the dynamical supersymmetry. First of all, we have the following boundary conditions

$$\partial_\tau X^{r'} = 0 = x_0^{r'}, \qquad (4.15)$$
$$\partial_\sigma X^i + \mathcal{F}^{ij}\partial_\tau X^j = 0. \qquad (4.16)$$

For $\widetilde{\Omega} = 1$, $X^a = 0$ by definition, so that the dynamical supersymmetry is maximally preserved. For $\widetilde{\Omega} = \Pi$ and $\gamma\Pi$, $\widetilde{\Omega}\Pi S^1 = S^1$ so that the supersymmetry is maximal if

$$\partial_\sigma X^a - \mu \cos\left(\frac{1}{2}\Theta_{jk}\right) X^a = 0. \tag{4.17}$$

This is the same kind of the boundary condition for the $(+,-,4,0)$, $(+,-,0,4)$ branes.

For the other branes, the dynamical supersymmetry can also be preserved by considering the spinor satisfying $\widetilde{\Omega}\Pi S^1 = \pm S^1$ at the boundary, but this time only 4 dynamical supersymmetries are preserved as was shown in [14] since the projection operator $\frac{1}{2}(1 \pm \widetilde{\Omega}\Pi)$ is now nontrivial. This case also requires the modified Neumann boundary condition like eq. (3.27) with the replacement $\mu \to \mu \cos(\frac{1}{2}\Theta_{jk})$. Note that the matrix $\widetilde{\Omega}$ corresponding to the $(+,-,1,1)$ and $(+,-,2,2)$ branes only can satisfy the condition (4.10) since the $(+,-,3,3)$ and $(+,-,4,4)$ branes cannot have additional Neumann directions satisfying eq. (4.13). Note that the brane position can be arbitrary when $\Theta_{jk} = 0$.

5. Supersymmetric D-branes in general background

Now we will relax the condition (4.15). First note that, as shown in the previous section, the projected spinors defined by

$$S^A_\pm \equiv \frac{1}{2}(1 \pm \widetilde{\Omega}\Pi) S^A \tag{5.1}$$

can only preserve the dynamical supersymmetry in the magnetic flux background. So we have to consider the spinor $S^1_\pm$ satisfying

$$\gamma^{ar'jk} S^1_\pm = \pm S^1_\pm \tag{5.2}$$

or

$$\gamma^{r's'jk} S^1_\pm = \pm S^1_\pm, \tag{5.3}$$

where the $\pm$ sign in the right-hand side is just an eigenvalue of the matrix $\gamma^{ar'jk}$ or $\gamma^{r's'jk}$ (independent of that in eq. (5.1)) and we will not concern the sign.

For the case (5.2), we have to further introduce a constant electric flux generated by the linear gauge field $A_+ = -F_{+a}X^a$ in addition to the quadratic piece eq. (3.27), where $F_{+a} = \mu x_0^{r'} \sin\frac{\Theta_{jk}}{2}$. In this case the Neumann boundary condition is given by

$$\partial_\sigma X^a - \mu \cos\frac{\Theta_{jk}}{2} X^a + \mu x_0^{r'} \sin\frac{\Theta_{jk}}{2} = 0. \tag{5.4}$$

The D-brane with $\widetilde{\Omega} = 1$ does not belong to the above case since $X^a = 0$ by definition. However, we can introduce two independent fluxes for the D-branes with $\widetilde{\Omega} = \Pi$ and $\gamma\Pi$ - at least 2 dynamical supersymmetries. For example, $\gamma^{ar'jk} = \gamma^{1756}$ or γ^{2856} for $\Omega = \Pi \exp^{\frac{1}{2}\Theta_{56}\gamma^{56}}$. The other cases allow only one independent projection, so that they also preserve 2 dynamical supersymmetries. For example, $\gamma^{ar'jk} = \gamma^{1237}$ or $\gamma^{1248} = -\gamma\widetilde{\Omega}\Pi\gamma^{1237}$ for $\Omega = \gamma^{3456}\exp^{\frac{1}{2}\Theta_{12}\gamma^{12}}$.

For the case (5.3), on the other hand, we need the modified Dirichlet boundary condition

$$\partial_\tau X^{r'} - \mu v^{s'} \sin \frac{\Theta_{jk}}{2} = 0, \tag{5.5}$$

where $v^{s'} \equiv X^{s'}|_{\partial\Sigma}$ for some $s' \in D$. This means that the D-brane is constantly boosted along the r'-direction. When $\Theta_{jk} = 0$, it satisfies the usual Dirichlet boundary condition, consistent with 3.2. The D-brane with $\widetilde{\Omega} = 1$ can have two independent boosts (or angular momenta) while the D-branes with $\widetilde{\Omega} = \Pi$ and $\gamma\Pi$ allow only one angular momentum, so that they preserve at least 4 dynamical supersymmetries. The other branes can have only one independent boost, so at least 2 dynamical supersymmetries are preserved.

In order to consider eq. (5.2) and eq. (5.3) simultaneously, we need at least three Dirichlet directions and $X^a \neq 0$. This is satisfied only by the brane, for example, with $\Omega = \gamma^{35} \exp^{\frac{1}{2}\Theta_{12}\gamma^{12}}$. This brane can preserve 2 dynamical supersymmetries since $\gamma^{ar'jk} = \gamma^{1236}$ and $\gamma^{s't'jk} = \gamma^{1278} = -\gamma\widetilde{\Omega}\Pi\gamma^{1236}$.

6. Discussion

We studied D-branes in the type IIB plane wave background together turning on additional backgrounds — electric as well as magnetic fluxes and an angular momentum. We found a much richer spectrum of supersymmetric D-branes compared to the flat spacetime. Let us briefly summarize the results obtained in this paper.

It turned out that the D_--branes and the D_+-branes behave very differently when an electric flux and an angular momentum are turned on. The D_--branes can be placed away from the origin by introducing a constant electric flux. So this process in general breaks a global world-volume symmetry except some special case. For a $(+,-,m,n)$ brane, the D-brane worldvolume theory has the global symmetry $\mathrm{SO}(m)\times\mathrm{SO}(n)\times\mathrm{SO}(4-m)\times\mathrm{SO}(4-n)$. The breaking of the symmetry $\mathrm{SO}(m)$ or $\mathrm{SO}(n)$ by the electric flux is necessarily correlated with that of $\mathrm{SO}(4-m)$ or $\mathrm{SO}(4-n)$ due to the shift of transverse position. One exception is a $(+,-,3,1)$ or $(+,-,3,1)$ brane which preserves the maximal supersymmetry as discussed in 3.1.1. In this case the electric flux and the transverse shift do not touch the global symmetry $\mathrm{SO}(3) \times \mathrm{SO}(3)$. D_+-branes, however, do not break any global symmetry by the electric flux. But, in this case, the electric flux is not constant but linearly proportional to Neumann coordinates. Note that the D_+-branes can take arbitrary transverse position without breaking supersymmetry [14]. We also observed that the D_--branes can also move with constant velocity preserving some amount of supersymmetries. However the D_+-branes cannot move while preserving the supersymmetry.

Since the oblique D-branes contain both D_--like and D_+-like supercharges, a similar feature also appears as the D_--branes. But, only if the electric flux is turned on to preserve the D_+-like supercharge, some OD-branes can then be shifted away from the origin after further introducing a constant electric flux or move with constant velocity, while preserving some supersymmetries.

We also considered the magnetic flux background. As observed by Mattik [15], we showed that the D-brane with magnetic flux is continuously interpolating from a D_+-brane to a D_--brane, in general, with different amount of supersymmetries at the endpoints. Our analysis shows that the OD-branes cannot preserve any dynamical supersymmetry in the magnetic flux background. We observed that there exist supersymmetric moving D-branes satisfying eq. (5.5). Note that these D-branes already carry the electric flux F_{+a} as well as the magnetic flux F_{jk}. These D-branes thus correspond to giant gravitons rotating in the $X^{r'} - X^{s'}$ plane with the nontrivial worldvolume gauge field. For example, the D-brane with $\Omega = \gamma^{35} \exp^{\frac{1}{2}\Theta_{12}\gamma^{12}}$ can preserve 2 dynamical supersymmetries with the nontrivial $F_{+3,5}$, F_{12} and L_{78} background. Recently this kind of giant graviton was found [50] in the $AdS_5 \times S^5$ geometry. It could be interesting to see whether the giant graviton in [50] after the Penrose limit can be reduced to that in the plane wave geometry.

In this paper we did not consider intersecting D-branes [51, 52]. It should be straightforward to extend the analysis in this paper to the intersecting D-branes following the scheme in [39].

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A. Matrix relations

Here we list useful matrix relations for the OD-branes with $\Gamma = \gamma^{1256}$ in table 1 which were used to find supersymmetric backgrounds in the subsection 3.3.

- $OD3$-brane with $\Omega = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 + \gamma^5)$:

$$\Omega\Pi P_+ = \gamma^{2345} P_+ \text{ or } -\gamma^{1346} P_+, \tag{A.1}$$

$$\Omega\Pi P_- = -\gamma^{34} P_- \text{ or } -\gamma\gamma^{78} P_-. \tag{A.2}$$

- $OD5$-brane with $\Omega = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 - \gamma^5)\gamma^{34}$:

$$\Omega\Pi P_+ = P_+, \tag{A.3}$$

$$\Omega\Pi P_- = \gamma^{25} P_- \text{ or } \gamma^{16} P_-. \tag{A.4}$$

- $OD5$-brane with $\Omega = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 - \gamma^5)\gamma^{78}$:

$$\Omega\Pi P_+ = -\gamma P_+, \tag{A.5}$$

$$\Omega\Pi P_- = \gamma\gamma^{25} P_- \text{ or } \gamma\gamma^{16} P_-. \tag{A.6}$$

- $OD5$-brane with $\Omega = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 + \gamma^5)\gamma^{37}$:

$$\Omega\Pi P_+ = -\gamma^{2457} P_+ \text{ or } \gamma^{1467} P_+ \tag{A.7}$$

$$\Omega\Pi P_- = -\gamma^{47} P_- \text{ or } -\gamma\gamma^{38} P_-. \tag{A.8}$$

- $OD7$-brane with $\Omega = \frac{1}{2}(\gamma^1 - \gamma^6)(\gamma^2 + \gamma^5)\gamma^{3478}$:

$$\Omega\Pi P_+ = -\gamma^{2578} P_+ \text{ or } -\gamma^{1678} P_+ \tag{A.9}$$
$$\Omega\Pi P_- = \gamma^{78} P_- \text{ or } \gamma\gamma^{34} P_-. \tag{A.10}$$

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